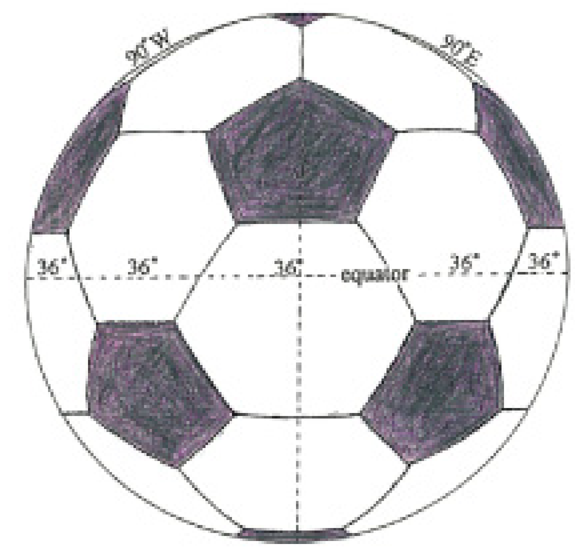
Sphero-geometry

When a polyhedron is insphered within a sphere with its vertices touching the sphere, there is an intimate relationship between the polyhedron and the sphere. This is because they both have a concentric centre and the diameter of the sphere is equal to the straight line distance from a vertex of the polyhedron to its diametrically opposite vertex, which also goes through the centre like a diameter. This is true of all insphered polyhedra including the hexpenhedron. The only exception is the tetrahedron, where the straight line from its vertex goes through the centre to touch the sphere, as there is no vertex diametrically opposite. In insphered polyhedra the vertices of the arced geometric figures rest on the vertices of the similar flat geometric figure under it. And the sides of the arced geometric figures are always aligned along great circles of the sphere. This is true with reference to the diagrams of the football and the hexpenhedron featured in the opposite page. In the composite diagram below, one is superimposed over the other to show the intimate relationship between the two in an insphered state.

Sphero-geometry is therefore the geometry of figures like arced triangles, squares, pentagons, hexagons and great circles that are projected by insphered polyhedra onto the surface of a sphere. The measurements of their sides and their areas are dependant on the circumference and superficial area of the sphere. The length of a side or arc is calculated from the circumference of the sphere with reference to the angle it supports at the centre of the sphere. Thus a side or arc of a sphero-geometric figure is sometimes referred to as an arc-angle with a specific angular value. Because a sphero-geometrical figure is arced the length of its side and its area is greater than that of a flat figure. The similarity of the two figures is determined by the fact that the arced figure rests on a flat figure as shown in the composite diagram of the insphered hexpenhedron. The area of a sphero-geometrical figure is calculated by dividing the superficial area of the sphere proportionately among the sphero-geometric figures that share the superficial area.

(Side is the compromise term I use for edge, to describe a common feature on a sphere and a polyhedron.)

A composite diagram of the hexpenhedron insphered within the football
(Note that the sides of the sphero-geometic hexagons and pentagons on the football are curved, and the sides of the hexpenhedron insphered within are straight)



Photographic evidence validating the ten 36° equatorial arc-angles
(Note that there are five complete 36° arc-angles along a half of the equator, making it ten for the whole equatorial circumference.)

Discovering the arc-angle value of a side of the insphered hexpenhedron

Consider the insphered hexpenhedron within the football. ABCDEF is a composite hexagon (an arced one superimposed onto a flat one). G,H and G_1,H_1 are the midpoints of the sides CD and FE, of the straight side and arced side, of the hexagon respectively. O is the concentric centre of the sphere (football) and the hexpenhedron. A light at the centre O could radiate and project GH onto G_1H_1.

By exploiting the intimate relationship between the hexpenhedron and the sphere in the insphered state, the following conclusions can be reached.

(1) The arc-angle value of G_1H_1 is 36° ($\frac{1}{10}$ arc of the equator). But G_1H_1 is a projection of GH. Therefore GH also supports an angle of 36° at the centre.

(2) If the linear value of GH is 36, then the linear value of FA, of the regular hexagon, is 24 (side of a hexagon equals $\frac{2}{3}$ the distance of the straight line connecting the midpoints of its alternate sides). The same value can be given in degrees for the angle FA supports at the centre. By transference, the arc-angle value of FA, a side of the arced hexagon, is also 24° . But FA is also a side of the adjacent pentagon of the hexpenhedron. Therefore by transference, any side of the insphered hexpenhedron has an arc-angle value of 24° .

Sphero-geometry can be used for instant measurements of distances and areas on a spherical surface. My objective is to use it to measure the earth which is imagined to be a perfect sphere. However, the differences due to the earth's oblate-spheroid shape has to be borne in mind. In the same manner as described for the insphered hexpenhedron, the arc-angles of the sides of the insphered tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron can be figured out, and the earth could be measured in a greater variety of ways.